1.1.1.1 Resonator responses on a Smith Chart

One key measurement made with Smith charts is the measurement of resonators to find the Q and center frequency. For 1-port resonators, the procedure to find the Q and center frequency is relatively simple. Consider the circuit of Figure 1. The key figures of merit for the resonator are \( f_0 \), the center frequency, and \( Q_0 \), the unloaded Q. The center frequency and Q can be found from

\[
f_0 = \frac{1}{2\pi \sqrt{LC}}, \quad Q_0 = 2\pi f_0 R_0 C
\]  

(0.1)

Figure 1 Schematic of a 1 port resonator with coupling capacitance

The common measurement of Q is to look for the 3-dB loss points of a transmission (S21) response, relative to the center frequency, where the loaded Q is defined as

\[
Q_L = \frac{f_0}{f_2 - f_1}
\]

(0.2)

And \( f_2 \) and \( f_1 \) are the frequencies of the lower and upper 3 dB down points. Loaded Q means that the circuit is loaded by the external resistance of the VNA source, typically 50 ohms. The usual figure of merit for a resonator is unloaded Q, or \( Q_0 \). More on these S21 based measurements will be discussed in Chapter 5

Here, the measurement is S11, so the normal definition doesn’t apply. When a high Q resonator is measured with a 1-port S11 measurement as in Figure 1, there may not be any point below 3 dB so the concept of looking for a magnitude response fails to find a Q value. When viewed on a Log magnitude return loss plot, the reflection from a high Q circuit is hard to see, as the return loss is very nearly 1 for all frequencies. Therefore, it is common to add a coupling structure, often a very small capacitance which transforms the impedance of the lossy elements to match the impedance of the test system. Figure 2 shows the return loss of a relatively low Q resonator with a direct connection and with a coupling capacitance added to match the circuit to \( Z_0 \). If the resonator was of a higher Q, the direct connection trace would
show almost no change in return loss and appear as a flat line of S11 nearly equal to 0 dB return loss.

When plotted on a Smith chart with a direct connection, the resonator forms a perfect circle and crosses the real axis at \( f_0 \). But in real world measurements, there is almost always some external transmission line that shifts the response of the resonator as shown in Figure 3. Also shown is the same resonator with the same value of coupling capacitance as used in Figure 2. From these Smith chart plots, the Q factor can be directly computed.

The center frequency is found by looking for the point where the trace crosses over itself. Drawing a line from this point through the center of the smith chart marks the position on the trace trajectory that represents the resonant frequency, \( f_0 \). From this line, a position is marked at \( \pm 45 \) degree angles, and a line drawn from the crossing point, at these angles, until it crosses the Smith chart trace at frequencies \( f_1 \) and \( f_2 \). One can use these frequencies to measure the loaded Q of the circuit as in (0.2). The diameter, \( d \), of the circle formed on the Smith chart is a measure of difference between loaded and unloaded Q, and the unloaded Q can be computed from

\[
Q_0 = Q_L \left[ 1 + \left( \frac{d}{2d - 1} \right) \right]
\]

(0.3)
Thus, measurements of unloaded Q can be made using the Smith chart trajectory without knowing anything about the value of the coupling capacitance. More details on Q measurements with a VNA will be discussed in Chapter 5.

Figure 3 Smith chart plot of a directly connected resonator and one matched to \( Z_0 \) using a coupling capacitance.